Modeling and Algorithm for Dynamic Multi-Objective Max-CSPs

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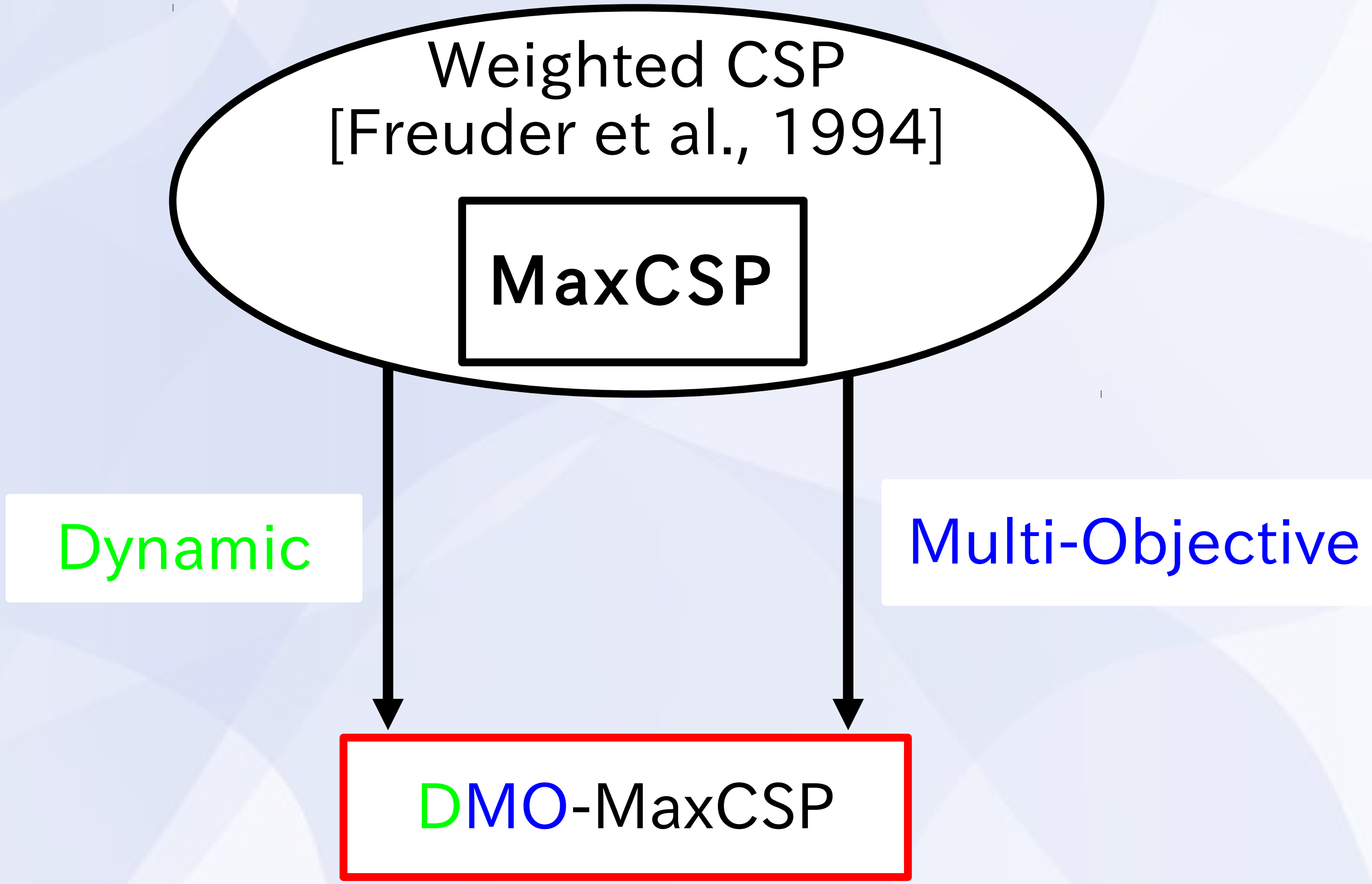
On going

- Complete/Incomplete algorithms for MO-DCOPs
  - Proof for generalisation
  - Comparision is required with B-MOMS
- AOF-technique based Algorithm for Dynamic MO-DCOPs
  - Diversity : method to provide weights
- Modeling and Algorithm for Dynamic MO-DCOPs
  - Reactive approach : everything can be changed (s-robustness)
- Modeling and Algorithm for Dynamic MO-Max-CSPs
  - Proactive approach: with Nicolas
  - Reactive approach: with Tony
- Prism problem (extension of WSR-paper with Ikegai)

Motivation

- Many real world problems involve multiple criteria that should be considered separately.
- Many real systems are dynamic, i.e., problems change at runtime.

Related Works



Summary

- Provide a model for DMO-MaxCSPs.
  - Defined by a sequence of MO-MaxCSPs.
  - # variables / domains / # constraints / # objectives can be changed.

Summary

- Provide a model for DMO-MaxCSPs.
  - Defined by a sequence of MO-MaxCSPs.
  - # variables / domains / # constraints / # objectives can be changed.
- Develop the first algorithm for DMO-MaxCSPs.
  - Find a sequence of a set of solutions based on new solution criteria l-, m- and s-robustness.
  - The complexity is bounded by the parameter s.

Outline

- Preliminaries
  - CSP/Max-CSP
- Multi-Objective Max-CSP :
  - (l,m)-robustness
- Dynamic MO-MaxCSP (DMO-MaxCSP)
  - s-weak robustness/(l,m,s)-robustness
- Algorithm for DMO-MaxCSPs
- Experiments
- Conclusion and future work

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CSP/Max-CSP

- CSP/Max-CSP is defined by <V, D, C> where
  - V : a set of variables,
  - D : a set of domains,
  - C : a set of constraints.
- Goal for CSP: Find a consistent assignment of values to variables.

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- Goal for CSP: Find a consistent assignment of values to variables.
- Goal for Max-CSP : Find an assignment which satisfies as many constraints as possible.

  Find an assignment which minimizes the number of violated constraints.

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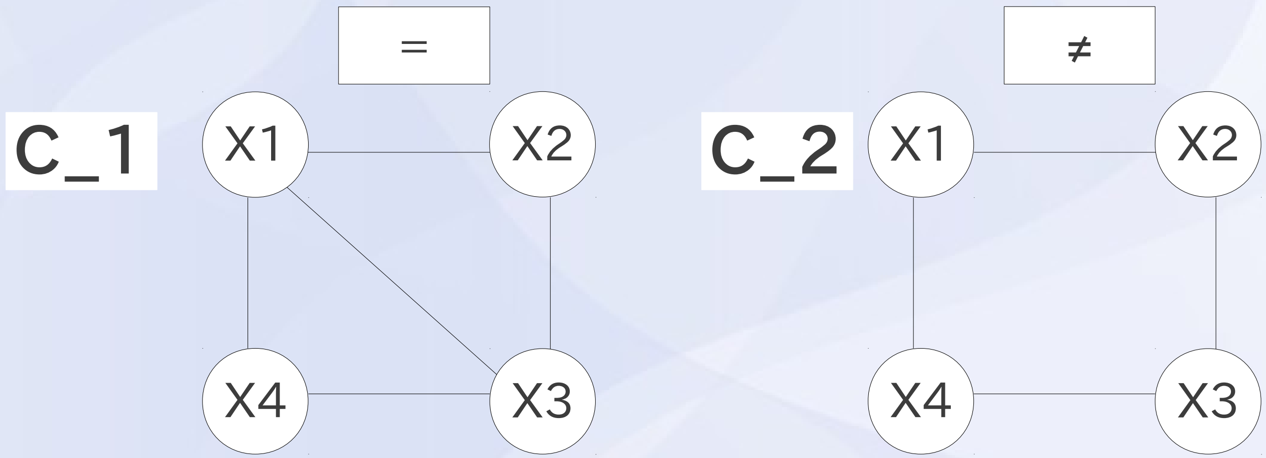
Multi-Objective Max-CSP (Formalisation)

- MO-MaxCSP is defined by <V, D, Γ> where
  - V : a set of variables,
  - D : a set of domains,
  - Γ : a set of sets of constraints {C_1,...,C_k} (k criteria/objectives which cannot merge)

- Solving an MO-MaxCSP is to find a Pareto front.

Multi-Objective Max-CSP (Example)



- Domain : {○,●}
- {(x1,○), (x2,●), (x3,○), (x4,●)} = (4,0)
- {(x1,○), (x2,○), (x3,○), (x4,○)} = (0,4)

Multi-Objective Max-CSP (Properties)

- For a cost vector R(A) and a constant l, we call that R(A)=(v_1,v_2,...,v_n) is a l-weak robust solution if each value v_i (1≤ i ≤ n) is blow l.
  - e.g. l=3 : No (5,0) / Yes (2,2)

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- For a cost vector R(A) and a constant l, we call that R(A)=(v_1,v_2,...,v_n) is a m-weak robust solution if v_1 + v_2 + ··· + v_n is blow m.
  - e.g. m=5 : No (3,3) / Yes (4,1)

Multi-Objective Max-CSP (Properties)

- For a cost vector R(A) and a constant l, we call that R(A)=(v_1,v_2,...,v_n) is a l-weak robust solution if each value v_i (1≤ i ≤ n) is blow l.
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- For a cost vector R(A) and a constant l, we call that R(A)=(v_1,v_2,...,v_n) is a m-weak robust solution if v_1 + v_2 + ⋯ + v_n is blow m.
  - e.g. m=5 : No (3,3) / Yes (4,1)
- R(A) is a (l,m)-robust solution if R(A) is l-weak robust solution and m-weak robust solution.
  - e.g. l=3, m=5 : No (4,1) / Yes (3,2)

Multi-Objective Max-CSP (Model)

- Given : MO-MaxCSP <V, D, Γ, l, m>
  - V : a set of variables,
  - D : a set of domains,
  - Γ : a set of sets of constraints {C_1,...,C_k},
  - l, m : constant
- Goal : Find a set of (l,m)-robust solutions.

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Dynamic MO-MaxCSP (Formalisation)

- DMO-MaxCSP is defined by a sequence of MO-MaxCSPs.

  <MO-MaxCSP_1,..., MO-MaxCSP_p>

- # variables/domains/# constraints/# objectives can be changed at run time.
- Goal: Find a sequence of a set of Pareto front.

  <PF_1,..., PF_p>

Dynamic MO-MaxCSP (Properties)

- Let R(A) and R(A') be two cost vectors of MO-MaxCSP and MO-MaxCSP'.
- s=dis(A',A): hamming distance between A and A'

  (# differences between two assignments).

Dynamic MO-MaxCSP (Properties)

- Let R(A) and R(A') be two cost vectors of MO-MaxCSP and MO-MaxCSP'.
- s=dis(A',A): hamming distance between A and A' (# differences between two assignments).
- R(A') is a s-weak robust solution if dis(A,A') ≤ s.
  - e.g. S=1 : No A=(○,●,○,●), A'=(○,○,○,○)
  - A=(○,●,○,), A'=(○,●,○,●) : dis(A,A')=0

Dynamic MO-MaxCSP (Properties)

- Let R(A) and R(A') be two cost vectors of MO-MaxCSP and MO-MaxCSP'.
- s=dis(A',A): hamming distance between A and A'

  (# differences between two assignments).
- R(A') is a s-weak robust solution if dis(A,A') ≤ s.
  - e.g. S=1 : No A=(○,●,○,●), A'=(○,○,○,○)
  - A=(○,●,○,), A'=(○,●,○,●) : dis(A,A')=0
- R(A') is a (l,m,s)-robust solution if A' is a (l,m)-robust solution and a s-weak robust solution.

Dynamic MO-MaxCSP (Model)

- Given : A sequence of MO-MaxCSPs

  <MO-MaxCSP_1,..., MO-MaxCSP_p> where

  MO-MaxCSP=<V, D, Γ, l, m, s>

- Goal: Find a sequence of a set of (l,m,s)-robust solutions.

  <RS_1,..., RS_p>

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Algorithm

- Based on BnB(+softAC)
  - Solve all (l,m,s)-solutions.
  - Pruning by criteria (l,m) and s.

Complexity

- Complexity:
  - $O(|PF| \bullet n \bullet k \bullet (|D\_max|^{S}))$ $(0 \leq s \leq d)$, where
  - d is the depth
  - n is the number of variables
- Parameters : k is the number of objectives.
  - $0 \leq l \leq n(n-1)/2$
  - $0 \leq m \leq k * \{n(n-1)/2\}$
  - $0 \leq s \leq n$

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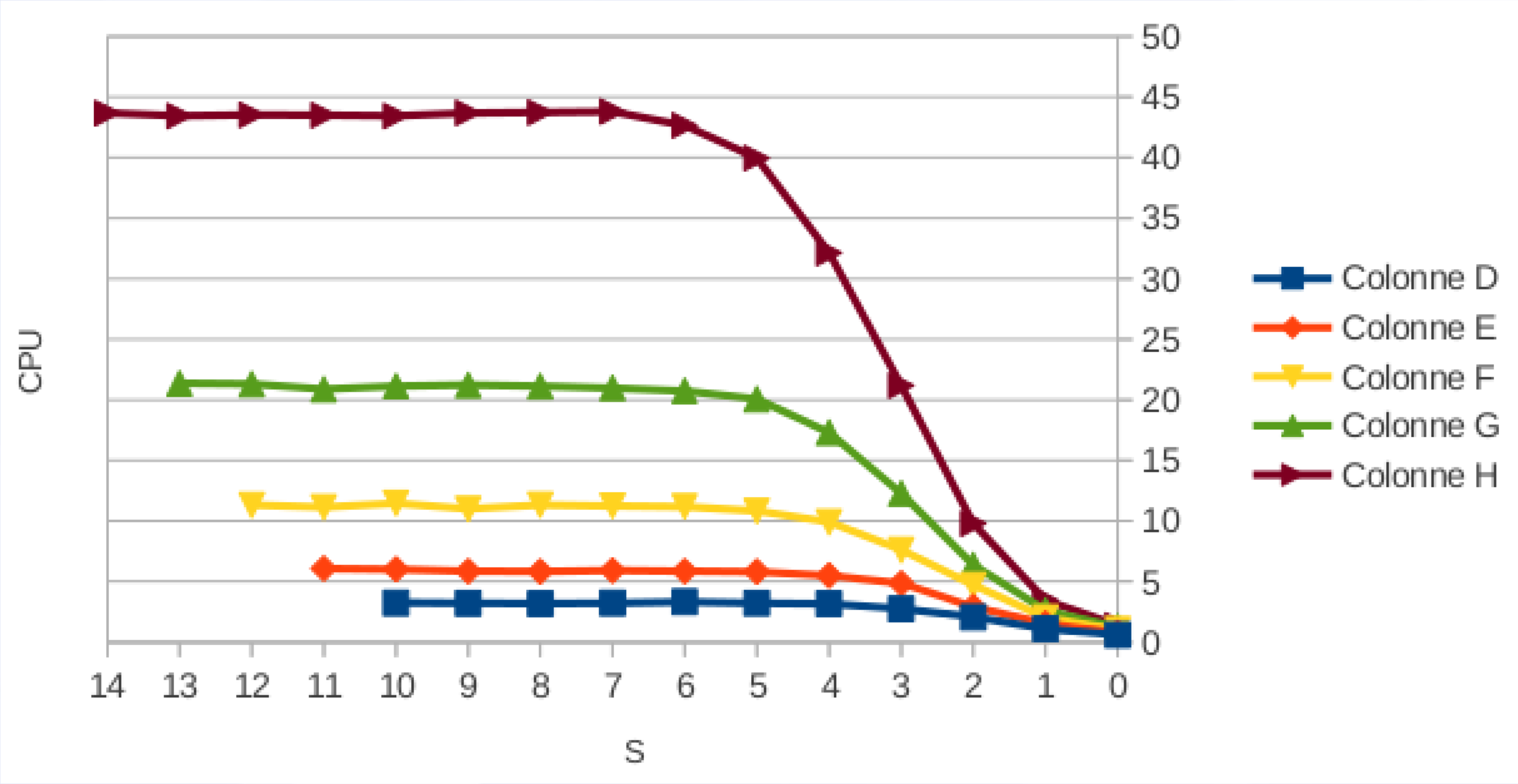
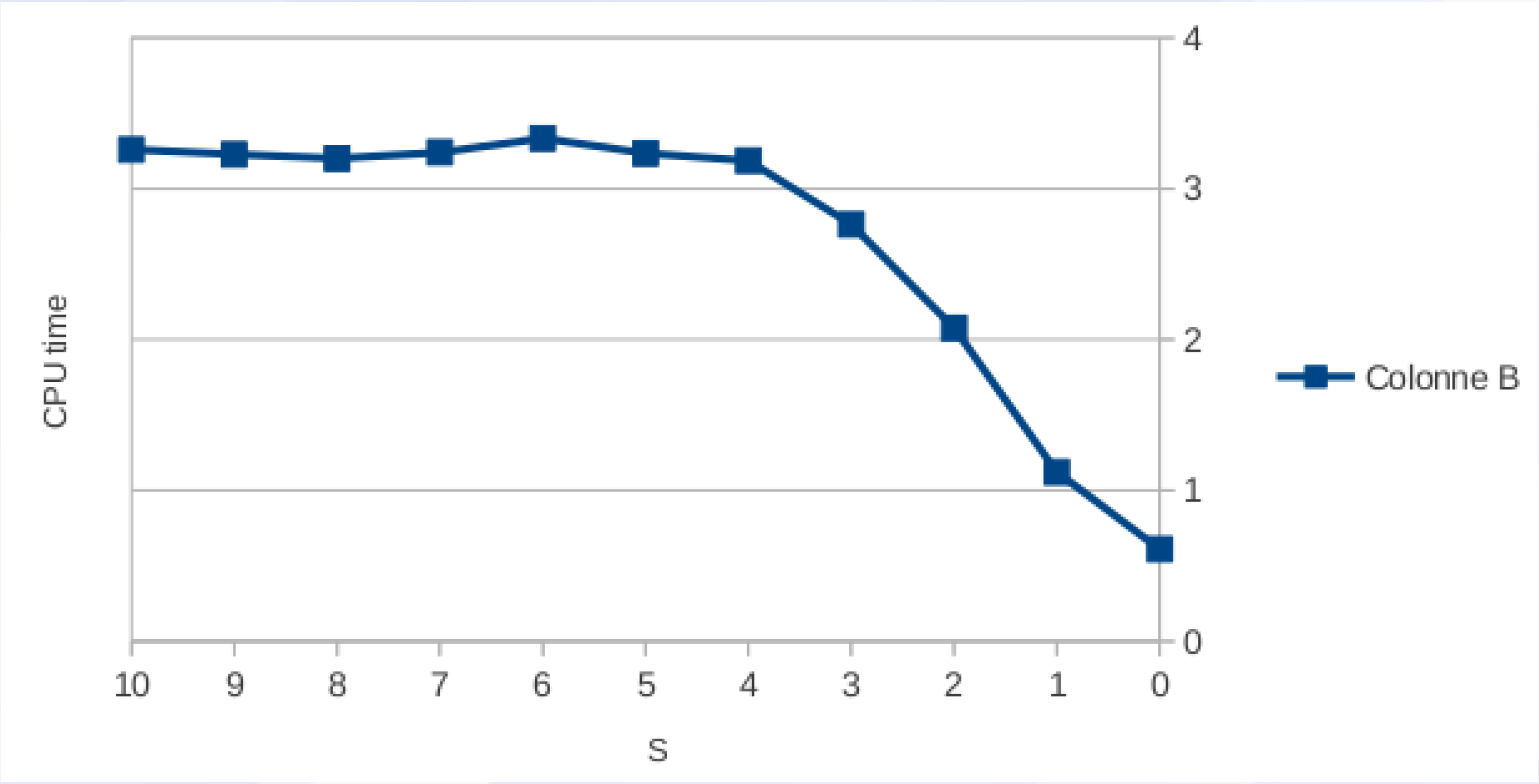
Experiments

- Objectives : 2->3
- Domain size : 2
- Problem instances : 50
- Varying the # nodes : 10-20

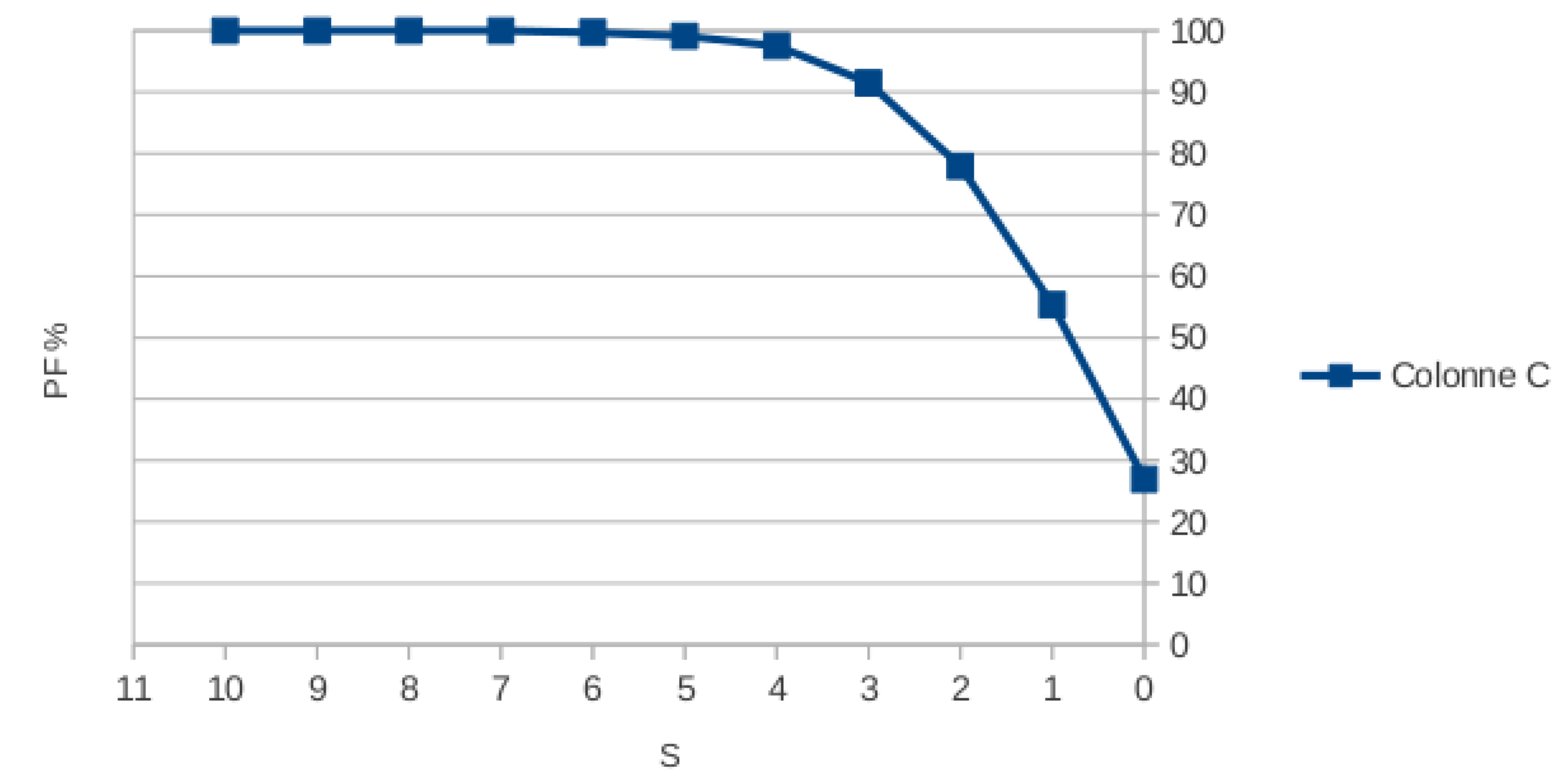
Different # nodes



Runtime (10 nodes)



Pareto front (10 nodes)



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Thanks!
and
Questions !

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